13. Suppose $x \propto y$ and $y \propto z$. When $x = 5$, $y = 22$. When $y = 7$, $z = 8$. Find $x : z$. (4 marks)

14. z varies directly as x^3 and inversely as y^2. Find the percentage change in z when x is decreased by 10% and y is doubled. (4 marks)

15. z varies directly as x and y^2. Find the percentage change in z when x is decreased by 5% and y is increased by 20%. (4 marks)

16. Suppose z varies directly as x and inversely as y^3. Find the percentage change in y when x is increased by 305% and z is increased by 20%. (4 marks)

17. The following table shows the relationship between the value $\$V$ of a machine and the number of hours h it operated in the past.

h	10	40	80	120
V	6000	1500	750	500

   (a) It is known that $V \propto h^n$, where n is an integer. Find the value of n.

   (b) Hence find the value of the machine which operated for 50 hours.

   (5 marks)

18. The weight W g of a wooden sphere is directly proportional to the cube of its radius r cm. It is known that a sphere of radius 8 cm weighs 409.6 g. If the wooden sphere is cut such that its radius is reduced to 6 cm, find the change in its weight. (4 marks)

19. The number of days t needed to complete a job is inversely proportional to the number of workers N. It is known that the job can be completed within 9 days if 12 workers participate.

   (a) Express t in terms of N.

   (b) If the job must be completed within 7 days, what is the minimum number of workers needed?

   (4 marks)

20. The volume V cm^3 of a metal cone varies directly as the square of its base radius r cm and its height h cm. Find the percentage change in the volume if the base radius is increased by 10% and the height is increased by 20%. (4 marks)

21. The time t seconds for the communication between two stations, station A and station B, varies directly as the distance d m between the two stations. When the two stations are 0.6 km apart, $t = 0.03$ s.

   (a) Express d in terms of t.

   (b) If station B is moved 0.1 km further away from A, find the extra time taken for communication.

   (4 marks)

Section A(2)

22. It is known that y is the sum of two parts. The first part varies directly as x and the second part varies directly as x^2. When $x = 1$, $y = 5$; when $x = 3$, $y = 21$.

   (a) Express y in terms of x. (3 marks)

   (b) Find y when $x = -1$. (1 mark)

   (c) Find x when $y = -4$. **Reference: HKDSE 13Q11** (2 marks)

For Q.23–Q.24, use the graph papers provided on P.337.

23. Suppose y varies directly as x^2 where $x > 0$. The following table shows some values of x and the corresponding values of y.

x	1	3	4	5	6
y	0.25	2.25	4	6.25	9

   (a) Find the variation constant. (3 marks)

   (b) Plot the graph of y against x for $1 \leqslant x \leqslant 6$. (2 marks)

   (c) From the graph, find x when $y = 2$. (Give the answer correct to 1 decimal place.) (1 mark)

24. N partly varies directly as M and partly varies directly as the square of M. The following shows some values of M and the corresponding values of N.

M	1	2	3	4	5	6
N	4	7	9	10	10	9

   (a) Express N in terms of M. (3 marks)

   (b) Plot the graph of N against M for $1 \leqslant M \leqslant 6$. (2 marks)

   (c) Hence find the value of M such that N attains its maximum value. Use the result to find the maximum value of N. (2 marks)

   Reference: HKCEE 01Q13

25. It is known that $x^2 : (y + 1) = 2 : 3$.

   (a) Find y if $x = 5$. (2 marks)

   (b) Cathy said that y is partly constant and partly varies as x. Is she correct? Explain your answer. (2 marks)

   (c) Find the value(s) of x such that $x = y$. (3 marks)

26. $f(x)$ is partly constant and partly varies directly as x^4. When $x = 3$, $f(x) = 130$; when $x = 4$, $f(x) = 480$.

   (a) Find $f(x)$. (3 marks)

   (b) Hence factorize $f(x)$. (3 marks)

27. Suppose that $(y + 3)$ is directly proportional to $\sqrt{x}$ where $x > 0$. When $y = 35$, $x = 16$.

   (a) Express x in terms of y. (3 marks)

   (b) If x is an integer, what is the least possible value of x when $y > 92$? (3 marks)

28. z is partly constant and partly varies inversely as x^2, where $x > 0$. When $x = 10$, $z = 8.5$; when $x = 15$, $z = 6$.

   (a) Express z in terms of x. (3 marks)

   (b) What happens to z when the value of x becomes very large? Explain your answer. (2 marks)

   (c) Sketch the graph of z against x for $x > 0$. (2 marks)

29. y is partly constant and partly varies directly as $(x + 1)^2$. When $x = -3$, $y = -48$; when $x = 2$, $y = -28$.

   (a) Express y in terms of x. (3 marks)

   (b) Show that y also varies jointly as $(x + 5)$ and $(x - 3)$. (2 marks)

   (c) Find the value of x when $y = 192$. (2 marks)

30. z varies jointly as x and y^2. When $x = 1$ and $y = 5$, $z = 100$.

   (a) Find y when $x = 2$ and $z = 8$. (3 marks)

   (b) If x decreases by 18% and z decreases by 10%, find the percentage change in y.
   (Give the answer correct to 3 significant figures.) (3 marks)

31. x varies inversely as y^3 and z varies inversely as y, where $x, y, z > 0$. It is known that when $x = 2$, $z = 4$.

   (a) Express x in terms of z. (4 marks)

   (b) If x decreases by 27.1%, find the percentage change in z. (3 marks)

32. $g(x)$ varies partly as x and partly as x^3. When $x = 2$, $g(x) = 24$; when $x = 3$, $g(x) = 21$.

   (a) Find $g(x)$. (3 marks)

   (b) Let $h(x) = g(x) + 45$.

     (i) Prove that $x - 5$ is a factor of $h(x)$.

     (ii) Hence factorize $h(x)$.

   Reference: HKCEE 05Q10 (4 marks)

33. The area A cm^2 of gold coating needed for a figurine is the sum of two parts, one part varies directly as the length of the figurine l cm, the other part varies directly as l^2, where $3 < l < 10$. When the length is 4 cm, the area is 16 cm^2, when the length is 8 cm, the area is 24 cm^2.

   (a) Express A in terms of l. (3 marks)

   (b) The area of gold coating for a new figurine is 21 cm^2. Find the length of the figurine. (3 marks)

   (c) The company produces a mini figurine which is similar to the new figurine for their customers as souvenir. If the length of the mini figurine is 2 cm, find the area of the gold coating. (2 marks)

   Reference: HKCEE 02Q11

34. The cost C of making a figure of volume V cm^3 is the sum of two parts, one part is a constant and the other part varies as V. When $V = 10$, $C = 600$; when $V = 16$, $C = 900$.

   (a) Find the cost of the figure when its volume is 23 cm^3. (4 marks)

   (b) There is a larger figure which is similar to the figure stated in **(a)**. It is known that the total surface area is 3 times more than the figure in **(a)**. Find the cost of making the larger figure. (2 marks)

   Reference: HKDSE 12Q11

35. The body mass index (BMI) x of a person varies jointly as his/her weight W kg and inversely as the square of his/her height h m. The BMI of a person of height 1.5 m and weight 54 kg is 24.

   (a) Express x in terms of h and W. (2 marks)

   (b) When $x \leqslant 18.5$, the person is considered to be underweighting; on the other hand, when $x \geqslant 25$, the person is considered to be overweighting.

     (i) For a person of height 170 cm, find his/her minimum weight such that he/she is overweighting.

     (ii) Tommy is 162 cm tall and his weight is 70 kg. Is Tommy overweighting? Explain your answer. (4 marks)

36. The cost C of making a cake is partly constant and partly varies inversely as the number of cakes N made. If 20 cakes are made, the unit cost is $17, if 30 cakes are made, the unit cost is $15.4.

   (a) Express C in terms of N. (3 marks)

   (b) If 40 cakes are made and they are sold at $25 each, find the total profit made. (2 marks)

   (c) Find the minimum number of cakes made such that the unit cost is at most $13. (2 marks)

37. A DVD rental store offers two schemes for their customers. The following table shows the details of the schemes:

Scheme A	Scheme B
Rental fee varies directly with the number of DVDs borrowed.	Rental fee is partly constant and partly varies directly with the number of DVDs borrowed.

   Frankie joined Scheme A and the rental fee for 5 DVDs is $60. On the other hand, Maggie joined Scheme B and the rental fees for 4 DVDs and 5 DVDs are $56 and $64 respectively.

   (a) Find the amount one has to pay if he/she joins Scheme B and borrows 7 DVDs. (3 marks)

   (b) Jacky has borrowed x DVDs and the rental fees are the same for both schemes. Find x. (3 marks)

   (c) The manager of the store wants to promote a "borrow 3 get 1 free" discount for both scheme. If a customer wants to borrow 9 DVDs, which scheme should he/she choose? Explain your answer. (2 marks)

38. A cannonball is launched vertically upwards from a cannon on the ground. The height h m of the cannonball from the ground at t seconds after launching is partly proportional to t and partly proportional to t^2. When $t = 4$ and $t = 7$, the cannonball is 80 m and 35 m above the ground respectively.

   (a) Express h in terms of t. (3 marks)

   (b) Find the time when the cannonball lands on the ground. (2 marks)

   (c) A helicopter is located 120 m vertically above the cannon. Can the cannonball hit the helicopter? Explain your answer. (3 marks)

Section B

39. It is known that X is the sum of two parts, one part is a constant and the other part varies directly as t.
When $t = 5$, $X = 11$; when $t = 2$, $X = 5$.

    (a) Find t when $X = 7$.   (4 marks)

    NF (b) If $X + 3 = \log y$, express y in terms of t.   (2 marks)

NF 40. y is partly constant and partly varies directly as $\log x$. When $x = 0.1$, $y = -1$; when $x = 100$, $y = 5$.

    (a) Express y in terms of x.   (3 marks)

    (b) It is known that $y \propto \log px^2$, where p is a constant. Using the result of **(a)**, find the value of p.
    (2 marks)

NF 41. P varies partly as $\sqrt{x}$ and partly varies inversely as y. When $x = 2$ and $y = 10$, $P = 18$. When $x = 8$ and $y = 16$, $P = 14$.

    (a) Express P in terms of x and y.   (3 marks)

    (b) Suppose $x = 18$ and $y = 20$. If y is increased by 20%, find the percentage change in x such that P remains unchanged. (Give the answer correct to 3 significant figures.)   (4 marks)

42. Suppose y varies partly as x and partly as $\dfrac{1}{x+2}$, where $x > 0$. When $x = 1$, $y = 0$; when $x = 3$, $y = 8$.

    (a) Express y in terms of x.   (3 marks)

    NF (b) If x is an integer such that $y < 20$, find the possible values of x.   Reference: HKCEE 04Q10   (4 marks)

NF 43. It is given that z varies jointly as 2^x and 3^y. When $x = 5$ and $y = -1$, $z = 224$.

    (a) Express z in terms of x and y.   (3 marks)

    (b) If $z = 21$, find $\dfrac{x}{y}$.   (4 marks)

NF 44. (a) Solve the equation $x^2 - 9x + 8 = 0$.   (2 marks)

    y is partly constant and partly varies as x^n, where n is an integer greater than zero. The following table shows some values of x and the corresponding values of y.

x	1	2	3	4	5
y	−8	20	96	244	488

    (b) By using the result in **(a)**, express y in terms of x.   (5 marks)

NF 45. It is known that $\log y$ is partly constant and partly varies directly as x.

    (a) Show that y varies directly as a^x, where a is a constant.   (3 marks)

    (b) Given that $a > 0$. If $y = 10$ when $x = 1$ and $y = 40$ when $x = 3$, express y in terms of x.   (3 marks)

    (c) Sketch the graph of $\log y$ against x.   (2 marks)

46. The cooling ability C units of a fan of size l cm is the sum of two parts, one part varies as l and the other part varies as the square of l. When $l = 30$, $C = 1200$; when $l = 40$, $C = 800$.

    (a) Express C in terms of l.   (3 marks)

    (b) At which size will the fan lose all of its cooling ability?   (2 marks)

    NF (c) Find the range of values of l when $C > 1050$.   (3 marks)

    NF (d) Find the greatest possible cooling ability and the corresponding size of the fan.   (3 marks)

Reference: HKCEE 06Q15

47. The time h hours required to finish a task is partly constant and partly varies inversely as the number of workers n. When $n = 10$, $h = 210$; when $n = 30$, $h = 170$.

    (a) Find the time required to finish the task if 5 workers participate.   (4 marks)

    (b) The planner of the task wants to speed up the progress such that it will be finished within 140 hours. Is it possible? Explain your answer.   (2 marks)

   The salary $\$S$ of each worker working on a task varies directly with the time h hours spent on the task. When a worker spends 170 hours on the task, he receives \$10 200.

    (c) (i) Express S in terms of h.

        (ii) If the total salary given to all workers working on a task is \$936 000, find the number of workers working on the task.
    (5 marks)

48. In the figure, fences are built around a square-shaped garden for protection. The cost $\$C$ for building the fence is the sum of two parts, one part is constant and the other part varies directly as the total length l m of the fence. When $l = 80$, $C = 8600$; when $l = 100$, $C = 10\,600$.
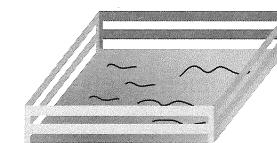
    (a) Express C in terms of l.   (3 marks)

    (b) What is the area of the square garden if the cost of setting up fences is \$7600?   (3 marks)

    (c) Find the cost in building the fence if the fence is 120 m long.   (2 marks)

    (d) If the cost in **(c)** is doubled, what is the percentage change in the area of the garden?   (3 marks)

49. (a) Let $f(x) = x^3 - 7x + 6$.

        (i) Show that $x - 1$ is a factor of $f(x)$.

        (ii) Hence, or otherwise, factorize $f(x)$.
    (4 marks)

    (b) Let $\$P$ be the profit in selling a certain product and w kg be the weight of the product. It is known that P varies partly as w^3 and partly as w. When $w = 10$, $P = 15\,810$; when $w = 5$, $P = 1530$.

        (i) Express P in terms of w.

        (ii) Find the weight of the product such that no profit or loss can be made by selling it.

        (iii) Using **(a)(ii)**, or otherwise, find the value(s) of w such that a loss of \$102 is made by selling it.
    (7 marks)